16/01

Dr. Ramjee Prasad (Mathematics), (Honours)

B.Sc. III. Title – Theory of Probability.

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(1). A bag contains 3 black and 4 red balls. Two balls are drawn at random one at a time without replacement. what is the first ball selected is black if the second ball is known to be red.

Solution. Let B_1 be the event of the first ball being black (bb, br) and B_2 be the event of the first ball being to find P(rb, rr).

Let A be the event of second ball being red i.e. (br, rr).

We are the find $P(B_1|A)$.

Now, See that A can happen with B_1 or B_2 i.e.

$$A = (A \cap B_1) \cup (A \cap B_2).$$

$$\therefore P(A \cap B_1) = P(B_1)\, P(A|B_1) = \frac{3}{7} \times \frac{4}{6} = \frac{2}{7}.$$

$$P(A \cap B_2) = P(B_2)\, P(A|B_2) = \frac{4}{7} \times \frac{3}{6} = \frac{2}{7}.$$

$$P(A) = P(A \cap B_1) + P(A \cap B_2) = \frac{2}{7} + \frac{2}{7} = \frac{4}{7}.$$

$P(B_1|A)$ = Required ~~Prog~~ Probability

$$P(B_1|A) = \frac{P(A \cap B_1)}{P(A)} = \frac{2/7}{4/7} = \frac{1}{2}.$$ Ans.

(2) There are 4 bags, each containing 6 white balls and 3 black balls and 3 bags each containing 2 white and 4 black balls. A ball is drawn, what is the chance for it coming from the first group?

Solution From 4+3 = 7 bags 4 belongs to the first group and 3 to the second group. Hence,

$$P_1 = \frac{4}{7}, \quad P_2 = \frac{3}{7}.$$

If a ball is selected from the first group the

Chance of drawing black ball is $\frac{3}{7}$ i.e. $\frac{1}{3}$.
If it is drawn from the second group, chance is $\frac{4}{6} = \frac{2}{3}$.

Thus, $P_1 = \frac{1}{3}$, $P_2 = \frac{2}{3}$.

Required Probability $= \frac{P_1 P_2}{P_1 P_1 + P_2 P_2} = \frac{\frac{1}{3} \times \frac{4}{7}}{\frac{1}{3} \times \frac{4}{7} + \frac{2}{3} \times \frac{3}{7}}$

$= \frac{2}{5}$ Ans.

(3) There are 3 bags and then contain 2 white and 3 black balls; 3 white and 2 black balls, 4 white and 1 black balls respectively. The Probability of selecting each ball is same. A bag is selected at random and a ball is drawn from it. (i) Find the chance that a white ball is drawn (ii) It is known that ball is white, what is probability that it come from second bag?

Solution Let A be the event of getting one white ball and B_1, B_2, B_3 be the events of selecting first, second and third bags respectively.

$P(B_1)$ = The Probability that the first bag is selected
$= \frac{1}{3}$.

Similarly, $P(B_2) = P(B_3) = \frac{1}{3}$.

$P(A/B_1)$ = The Probability that a white ball is drawn ~~white~~ while first bag is selected.

$= \frac{^2C_1}{^5C_1} = \frac{2}{5}$.

Similarly, $P(A/B_2) = \frac{3}{5}$

and $P(A/B_3) = \frac{4}{5}$.

(i) $P(A)$ = The probability that white ball is drawn

$$= \sum_{i=1}^{3} P(B_i)\,P(A|B_i)$$

$$= P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3).$$

$$= \frac{1}{3}\times\frac{2}{5} + \frac{1}{3}\times\frac{3}{5} + \frac{1}{3}\times\frac{4}{5}.$$

$$= \frac{1}{3}\left(\frac{2}{5}+\frac{3}{5}+\frac{4}{5}\right)$$

$$= \frac{1}{3}\cdot\left(\frac{9}{5}\right) = \frac{3}{5}$$

(ii) $$P(B_2|A) = \frac{P(B_2)P(A|B_2)}{\sum_{i=1}^{3} P(B_i)P(A|B_i)}$$

$$= \frac{\frac{1}{3}\times\frac{3}{5}}{\frac{1}{3}\times\frac{2}{5}+\frac{1}{3}\times\frac{3}{5}+\frac{1}{3}\times\frac{4}{5}}$$

$$= \frac{\frac{1}{5}}{\frac{3}{5}} = \frac{1}{5}\times\frac{5}{3} = \frac{1}{3}.$$

(4) There are three boxes containing respectively 1 white, 2 red, 3 black balls; 2 white, 3 red, 1 black balls and 3 white, 1 red 2 black balls. A box is chosen at random and from it two balls are drawn at random. The two balls are one red and one white. What is the probability that they come from the (i) first box (ii) second ~~ball~~ box (iii) third box.

Solution: Let A be the event of getting two balls one red and one white and let B_1, B_2, B_3 be the events of being first box, second box and third box.

$\therefore P(B_1)$ = the probability that the first box is selected

$= \frac{1}{3}$.

$P(B_2)$ = The probability that the second box is selected

$= \frac{1}{3}$.

$P(B_3)$ = the probability that the third box is selected

$= \frac{1}{3}$.

$\therefore P(A|B_1)$ = the probability that two balls one red and one white are coming from first box.

$$= \frac{{}^{1}C_1 \times {}^{2}C_1}{{}^{6}C_2} = \frac{2}{15}.$$

Similarly, $P(A|B_2) = \frac{{}^{2}C_1 \times {}^{3}C_1}{{}^{6}C_2} = \frac{2}{5}$.

$$P(A|B_3) = \frac{{}^{3}C_1 \times {}^{1}C_1}{{}^{6}C_2} = \frac{1}{5}.$$

Now, $P(B_1|A)$ = the probability that the first box is selected and two balls are one red and one white.

By Baye's Theorem we get

$$P(B_1|A) = \frac{P(B_1) \cdot P(A|B_1)}{P(B_1) P(A|B_1) + P(B_2) P(A|B_2) + P(B_3) \cdot P(A|B_3)}$$

$$= \frac{\frac{1}{3} \times \frac{2}{15}}{\frac{1}{3} \cdot \frac{2}{15} + \frac{1}{3} \cdot \frac{2}{5} + \frac{1}{3} \cdot \frac{1}{5}} = \frac{2}{11}.$$

$$P(B_2|A) = \frac{P(B_2) \cdot P(A|B_2)}{P(B_1) P(A|B_1) + P(B_2) \cdot P(A|B_2) + P(B_3) \cdot P(A|B_3)}$$

$$= \frac{\frac{1}{3} \cdot \frac{2}{5}}{\frac{1}{3} \cdot \frac{2}{15} + \frac{1}{3} \cdot \frac{2}{15} + \frac{1}{3} \cdot \frac{1}{5}} = \frac{6}{11}.$$