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- **Emissive Power:** If an isotropic body at T K emits per second per unit area an amount of energy e_λ in the form of radiation of wavelength λ and $\lambda + d\lambda$ into a vacuum then, e_λ is known as the emissive power of that body at that temperature for radiation of wavelength λ.

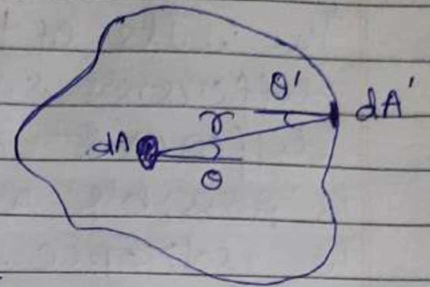
- **Absorptive Power:** If a given amount of radiation of wavelength between λ and $\lambda + d\lambda$ is incident from a vacuum on an isotropic body at T K, then the fraction a_λ of this energy that is absorbed by the body is called the absorptive power of the body at that temperature for radiation of wavelength λ.

- **Kirchoff's Law:** It states that the ratio of the emissive power to the absorptive power for radiation of a given wavelength is the same for all bodies at the same temperature and is equal to the emissive power of a perfect black body at that temperature.

Therefore, $\dfrac{e_\lambda}{a_\lambda} = \text{const} = E_\lambda$.

Derivation of Kirchoff Law :

Consider an enclosure at a uniform temp. T having walls opaque to radiations of all wavelengths and insulated thermally from the surroundings. The enclosure is filled with temperature radiation emitted by the walls. Consider a body A of emissive power e_λ and absorptive power a_λ being placed inside the enclosure at a large distance from the walls



(i) Whatever be the initial temperature of the body, it will ultimately attain the temperature T of the enclosure. For in the equilibrium state, the entropy becomes maximum which happens only when the temperature differences vanish.

(ii) Since any body placed inside finally attains the const temperature T of the enclosure, a heterogeneous body having different emissive and absorptive powers in different parts, if placed within the enclosure, the total energy absorbed by it will be equal to the total energy it emits, in the equilibrium state. Thus the total energy absorbed by the body is independent of its position or orientation with respect to the walls of the enclosure. As the different surfaces of the body have different absorption coefficients, this is possible when the radiation inside is isotropic.

The amount of energy emitted by an elemental area dA of the body in the direction between θ, $\theta + d\theta$ and ϕ, $\phi + d\phi$ is

$$e_\lambda \, d\lambda \, dA \cos\theta \sin\theta \, d\theta \, d\phi .$$

$\therefore$ The amount of energy emitted by dA for all wavelengths and for all possible values of θ and ϕ is

$$dA \int_0^\infty e_\lambda d\lambda \int_0^{\pi/2} \sin\theta \cos\theta \, d\theta \int_0^{2\pi} d\phi = \pi \, dA \int_0^\infty e_\lambda d\lambda$$

For the whole body, the emission is given by

$$\pi \left(\sum dA\right) \int_0^\infty e_\lambda d\lambda \quad \text{——} \quad (a)$$

But the amount of energy emitted by an elemental area dA' of the enclosure in the direction dA of the body is

$$dQ_\lambda = E_\lambda \, d\lambda \, dA' \cos\theta' \, \frac{dA \cos\theta}{r^2}$$

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where r is the distance betwee dA' and dA.
The amount of energy absorbel by dA of the body is.

$$a_\lambda \, dQ_\lambda = a_\lambda E_\lambda d\lambda \, dA \cos\theta \frac{dA' \cos\theta'}{r^2}$$

$$= a_\lambda E_\lambda d\lambda \, dA . \cos\theta . d\omega$$

$$= a_\lambda E_\lambda d\lambda \, dA . \cos\theta \, (\sin\theta \, d\theta \, d\phi)$$

where $d\omega = dA' \cos\theta' / r^2$ = solid angle substended by dA' at $dA = \sin\theta \, d\theta \, d\phi$.

∴ Total energy absorbed by dA from the total surface of the enclosure for all wavelengths is

$$dA \int_0^\infty a_\lambda E_\lambda d\lambda \int_0^{\pi/2} \cos\theta \, \sin\theta d\theta \int_0^{2\pi} d\phi = \pi \, dA \int_0^\infty a_\lambda E_\lambda d\lambda$$

Since the limits of integration extend for all wavelengths from 0 to ∞ and the value of the second integral is $\pi/2$ and that of the third 2π.
For the whole body, the absorption is

$$\pi \left(\sum dA \right) \int_0^\infty a_\lambda E_\lambda d\lambda \quad \text{——— (b)}$$

In the equilibrium state we must equate (a) and (b).

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$$\therefore \int_0^\infty e_\lambda \, d\lambda = \int_0^\infty a_\lambda E_\lambda \, d\lambda.$$

Now this equality must hold for each portion of the spectrum, ie for any value of λ.

Hence for each wavelength

$$e_\lambda = a_\lambda E_\lambda.$$

If the same body A is placed in another enclosure, at the same temp. T and with E'_λ emissive power, but having different shape and nature of the walls, then

$$e_\lambda = a_\lambda E'_\lambda$$

Since, e_λ, a_λ depends on the nature of the body and its temperature only

$$E_\lambda = E'_\lambda$$

If we place a black body having $a_\lambda = 1$ inside the enclosure, we have

$$E_\lambda = e_\lambda.$$

Thus, radiation in any hollow enclosure is independent of the nature and shape of the walls and is identical with black body radiation at that temperature.